MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LI. DECEMBER 12, 1890. No. 2

Lieut.-General J. F. TENNANT, C.I.E., R.E., F.R.S., President, in the Chair.

The Rev. John Mitchell, B.D., 57 Parkgate Road, Chester, was balloted for and duly elected a Fellow of the Society.

Lewis Boss, Dudley Observatory, Albany, N.Y., U.S.A.;
A. Cornu, Paris; and
C. Souillart, Lille,

were balloted for and duly elected Associates of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

George Price Blackwood Hallowes, Surveyor of the Post Office, 77 Lansdowne Road, Didsbury, Manchester (proposed by E. J. Spitta);

Shin Hirayama, Astronomer at the Tokyo Observatory, Japan (proposed by W. H. M. Christie);

Robert John Pearce, M.A. Cantab., Hon. D.C.L. Durham, Professor of Mathematics in Durham University (proposed by the Rev. T. E. Espin);

John M. Stone, St. John's College, Cambridge (proposed by the Rev. A. Wrigley).

Seventy-two Presents were announced as having been received since the last meeting, including, among others,

Ahmed Mukhtar Pasha, The Reform of the Calendar (in the original Turkish), presented by Sir J. W. Redhouse.

Celestial photographs taken at the Sydney Observatory, presented by H. C. Russell.

Observations of the Spectra of Sun-spots in the Region B—D made at the Stonyhurst College Observatory in the years 1882–89. By the Rev. A. L. Cortie, S.J.

(*Communicated by E. W. Maunder.*)

(*Abstract.*)*

The paper contains the results derived from the detailed study of the spectra of ninety Sun-spots in the region B—D. The observations were made after the plan followed at Greenwich, and the instrument employed was the Browning automatic spectroscope, generally with a dispersion of twelve prisms of 60° attached to the 8-inch equatoreal.

In the discussion the observations are divided into two periods, the first embracing the disturbed period of solar activity from 1882 till the autumn of 1886; and the second, called the quiet period, that from the autumn of 1886 till June 1889.

The widening of the lines has, as far as possible, been reckoned in tenths of the normal breadth of the lines. For the sake of comparison of the observations among themselves, two standards have been chosen, and a line widened between the limits 0·5 and 1·0 of its normal breadth is called a "more widened line," and that widened 1·0 or more a "most widened line." Lines below 0·5 are simply "widened." A long table gives the mean widening of nearly three hundred lines between B and D, both for the disturbed and the quiet period. Any interesting points about the lines are noted in a column headed "Remarks."

The main conclusions deduced from the discussion of these observations may be summarised as follows:—

1. The general absorption due to a spot occasionally varies in intensity in different portions of the spectrum. Two or three times it has been so dark at the red end as to partially mask the selective absorption of the lines. The spot-bands observed in 1885 and the first half of 1886 have not been seen in any spot observed from the autumn of 1886 to the autumn of 1890.

2. Of fifty-three iron lines occurring in this part of the spectrum, in the disturbed period only one had a mean widening placing it among the more widened lines.

3. In a total of 1,228 observations, the large majority having been taken in the disturbed period, only three in the disturbed period, and fourteen in the quiet period, placed iron lines among the most widened lines.

4. In the quiet period many more iron lines appeared among the more widened lines than in the disturbed period. Comparing the totals, 126 observations in 1885, and 127 in 1889, only three times in the former year was iron among the more widened lines, against eighty-one times in the latter year.

* To be published in the *Memoirs*, vol. l.

5. In general the iron lines seen bright in the chromosphere by Young have been least affected in Sun-spots.

6. There is a great variation in the lines of this metal which are selected for widening by Sun-spots, either by different spots, or by the same spot on different days and at different appearances.

7. Excepting four out of the eleven titanium lines in this part of the spectrum, the rest are very much affected in spots both at the maximum and the minimum epochs.

8. The lines most persistently affected are among the faintest of the Fraunhofer lines, and among the brighter of the metallic lines.

9. A characteristic of some titanium lines is that they are very marked in the spectrum of the penumbra of spots.

10. Only three lines of this element have coincident chromospheric lines, and two of these are among the least widened of the lines.

11. Including "basic" lines, or lines with the dispersion used common to two elements, of twelve lines of calcium observed, six have coincident bright lines in the chromosphere. While "basic" lines of calcium, which are coincident with bright lines, have been recorded among the most widened lines, simple calcium lines having coincident chromospheric lines have not entered into this class.

12. In the minimum period the mean widening of calcium has increased, but the general variation is not at all great.

13. Sodium, represented by four lines, which include the D lines, was very much affected in the large spots of the maximum epoch.

14. Lines of barium, nickel, and manganese have never entered among the most widened lines.

15. The short lines in the metallic spectra of barium and nickel have been more widened than the long lines.

16. Eleven "basic" lines occur in this region. In a total of 407 observations, only fifteen times have such lines been recorded among the most widened lines, and twelve of these instances are due to lines which have calcium as one component. Their appearance among the more widened lines is about 31 per cent.

17. With regard to lines of unknown origin, the following conclusions have been derived from 2,088 individual observations:—

(*a*) About the maximum period a great number of faint lines not in Ångström are to be seen in Sun-spots.

(*b*) Such lines are not seen exclusively in maximum spots, but reappear in minimum spots when they are large—*e.g.* in the fine spot of June 1889, and to a less extent in that of May 6 of the same year.

(*c*) Some faint lines which have been persistently watched are to be seen greatly widened in every Sun-spot, large or small, whether in the disturbed or quiet period.

(*d*) The mean widening of all the five bright chromospheric lines coincident with unknown lines in this region has been low.

18. The C line is generally, though by no means always, less dark or unaffected over Sun-spots. When reversed, the reversal has been generally due to the faculæ between the spots.

19. Several lines marked as "telluric" in Ångström's map have been widened in Sun-spots.

20. Taking all the observations together, the lines in this region might provisionally be arranged as affected in Sun-spots in the following order:—Lines of unknown origin, faint lines of titanium, lines of calcium, sodium, iron, barium, nickel, and manganese.

On the Correction of Micrometric Measures for Refraction.

By Harold Jacoby, B.A.

(*Communicated by Dr. Gill.*)

The following investigation was undertaken in consequence of a suggestion made by Dr. David Gill, H.M. Astronomer at the Cape of Good Hope. Dr. Gill pointed out to me that the reduction of stellar parallax observations would be much facilitated if the refraction correction were expressed in terms of the true distance and position angle, and the formulæ carried to a degree of approximation high enough for the accurate correction of large distances.

Modern micrometers are capable of measuring much greater distances than could be managed with Bessel's heliometer; and it is therefore necessary to retain certain terms in the development of the refraction which were rejected by Bessel as insignificant. It is hoped, however, that their retention will entail but little additional labour upon the computer, and that the numerical work will be considerably lessened by the use of the tables which I have computed. These tables will also be found quite convenient for the correction of distance and position angle measures made with the ordinary equatoreal; for it will then be sufficient to use Table I. for the mean refraction, and to apply a correction, depending on the state of the meteorological instruments, easily derived from Table II. The method of doing this will be explained further on. It is, of course, needless to add that the tables are also adapted to the correction of stellar parallax or other measures made by the photographic method.

I shall first examine the corrections necessary for distance measures. If we assume the following notation:—

δ_0, ζ_0 = the means of the true declinations and zenith distances of the stars observed.

l_0, λ_0 = the apparent and true angles which the great circle joining the two stars makes with the vertical circle through the point midway between them.

s, σ = the apparent and true distances of the stars.

α^* = the usual Besselian quantity of vertical refraction.

α'' = the usual Besselian quantity for correcting micrometric measures for refraction.

p, π = the apparent and true position angles.

q = the true parallactic angle, corresponding to the zenith distance ζ_0.

ζ, ζ' = the true zenith distances of the two stars.

r, r' = the corresponding vertical refractions.

r_0 = the refraction corresponding to ζ_0.

we shall obtain, in a manner quite analogous to the method given by Chauvenet: †

$$l_0-\lambda_0=\frac{b-a}{b+a}\sin 2\lambda_0+\tfrac{1}{2}\left(\frac{b-a}{b+a}\right)^2\sin 4\lambda_0+\ldots. \qquad (1)$$

$$s-\sigma=-\sigma\left\{\frac{a-1}{a}+\frac{b-a}{ab}\cos^2\lambda_0-\frac{1}{2a}\left(\frac{b-a}{b+a}\right)^2\sin^2 2\lambda_0+\ldots.\right\} \qquad (2)$$

in which

$$a=\frac{\sin\zeta_0}{\sin(\zeta_0-r_0)} \qquad (3)$$

$$b=\frac{\zeta-\zeta'}{\zeta-\zeta'-(r-r')} \qquad (4)$$

Chauvenet, following Bessel, puts

$$r-r'=\frac{dr_0}{d\zeta_0}(\zeta-\zeta'),$$

and has, therefore, from (4)

$$b=\frac{1}{1-\dfrac{dr_0}{d\zeta_0}}.$$

But, in order to secure sufficient accuracy, it will be necessary to write:

$$r-r'=\frac{dr_0}{d\zeta_0}(\zeta-\zeta')+\tfrac{1}{2}\frac{d^2r_0}{d\zeta_0^2}(\zeta-\zeta')^2,$$

or, for brevity,

$$r-r'=\frac{dr_0}{d\zeta_0}(\zeta-\zeta')+n(\zeta-\zeta')^2,$$

and, consequently, in equation (4) the denominator will require a correction of

$$n(\zeta-\zeta')^2,$$

* See Chauvenet, *Spherical and Practical Astronomy*, vol. ii. Table II.

† *Spherical and Practical Astronomy*, vol. ii. p. 452. See also Engelmann, *Abhandlungen von F. W. Bessel*, vol. iii. p. 343.

and therefore b will require a correction of

$$+n(\zeta-\zeta'),$$

or, with sufficient accuracy,

$$+n\sigma\cos\lambda_0;$$

and since

$$\lambda_0=\pi-q$$

the correction required by b becomes

$$+n\sigma\cos(\pi-q).$$

This small correction required by Bessel's b will produce an appreciable effect on the refraction correction at large zenith distances. I shall disregard it for the present, but will show afterwards how it can be taken into account.

In the case of a no appreciable correction will be necessary on account of putting, as Bessel did,

$$r_0=\tfrac{1}{2}(r+r').$$

We shall have then, again following Chauvenet,

$$a=1+\alpha$$
$$b=1+\alpha+\alpha''\tan^2\zeta_0$$

and therefore, including terms of the second order,

$$\frac{a-1}{a}=\alpha(1-\alpha)$$

$$\frac{b-a}{ab}=\alpha''\tan^2\zeta_0(1-2\alpha-\alpha''\tan^2\zeta_0)$$

$$\frac{1}{2a}\left(\frac{b-a}{b+a}\right)^2=\tfrac{1}{8}\alpha'^2\tan^4\zeta_0,$$

which, substituted in equation (2), give:

$$s-\sigma=-\sigma[\alpha(1-\alpha)+\alpha''\tan^2\zeta_0\cos^2\lambda_0(1-2\alpha-\alpha''\tan^2\zeta_0)$$
$$-\tfrac{1}{8}\alpha''^2\tan^4\zeta_0\sin^2 2\lambda_0].$$

In the second term of this equation we can put α'' for α, since the error caused thereby is only

$$2\alpha''\tan^2\zeta_0\cos\lambda_0(\alpha-\alpha''),$$

which cannot exceed $0''{\cdot}003$ at 80° zenith distance.

And if we also put

$$k_1=\frac{\alpha''}{1+\alpha''(1+\sec^2\zeta_0)}=\alpha''[1-\alpha''(1+\sec^2\zeta_0)] \qquad (5)$$

we obtain:

$$s-\sigma=-\sigma[\alpha(1-\alpha)+k_1\tan^2\zeta_0\cos^2\lambda_0-\tfrac{1}{8}\alpha''^2\tan^4\zeta_0\sin^2 2\lambda_0] \qquad (5a)$$

Now let

$$M = k_1 \tan^2 \zeta_0 \cos^2 \lambda_0, \tag{6}$$

and we have

$$s - \sigma = -\sigma\,[\alpha(1-\alpha) + M - \tfrac{1}{2}\left(\frac{\alpha''}{k_1}\right)^2 M^2 \tan^2 \lambda_0]. \tag{7}$$

From the refraction tables we find:

Maximum value of α'' 0·00026	Log 6·4158
„ „ α 0·00028	6·4487.

We may therefore neglect $\sigma\alpha^2$, whose maximum value for a distance of 8000″ is 0″·0006. Moreover, $\left(\frac{\alpha''}{k_1}\right)^2$ can never differ from unity by more than 0·017; and if we substitute unity for it, the error in the refraction for a distance of 8000″ cannot exceed 0″·0055. Making these indicated changes, and also putting

$$\lambda_0 = \pi - q,$$

equations (6) and (7) become

$$M = k_1 \tan^2 \zeta_0 \cos^2 (\pi - q) \tag{8}$$

$$s - \sigma = -\sigma\,[\alpha + M - \tfrac{1}{2} M^2 \tan^2 (\pi - q)]. \tag{9}$$

To return to the small correction required by b. We have, since

$$b - a = a'' \tan^2 \zeta_0,$$

the correction required by a'' equal to

$$\frac{\text{correction of } b}{\tan^2 \zeta_0},$$

or

$$\frac{n\sigma \cos (\pi - q)}{\tan^2 \zeta_0};$$

and from the usual Besselian formula,

$$s - \sigma = -\sigma\alpha''\{\tan^2 \zeta_0 \cos^2 (p - q) + 1\},$$

we see that the correction required by $s - \sigma$ will be given with sufficient accuracy by

$$\text{correction} = -\frac{n\sigma^2 \cos (\pi - q)}{\tan^2 \zeta_0}[\tan^2 \zeta_0 \cos^2 (\pi - q) + 1]$$

$$= -n\sigma^2 [\cos^3 (\pi - q) + \cos (\pi - q) \cot^2 \zeta_0].$$

To obtain the value of n, we have:

$$r_0 = \alpha' \tan \zeta_0,$$

where α' is the usual quantity for vertical refraction, with the argument "true zenith distance." Differentiating this equation, we have:

$$\frac{dr_0}{d\zeta_0} = \alpha' \sec^2 \zeta_0 \sin 1'' + \tan \zeta_0 \frac{d\alpha'}{d\zeta_0}, \tag{10}$$

by means of which a value of $\frac{dr_0}{d\zeta_0}$ may be computed for various zenith distances, the value of $\frac{da'}{d\zeta_0}$ being derived from the vertical refraction tables.

Now, in the equation

$$r - r = \frac{dr_0}{d\zeta_0}(\zeta - \zeta') + n(\zeta - \zeta')^2,$$

suppose

$$\zeta - \zeta' = 7200'.$$

We shall then have

$$\zeta = \zeta_0 + 3600'$$
$$\zeta' = \zeta_0 - 3600'.$$

With these values of ζ and ζ' we can derive r and r' from the vertical refraction tables; and the value of $\frac{dr_0}{d\zeta_0}$ having just been obtained, we can compute n thus: *

$$n = \frac{r - r' - 7200\frac{dr_0}{d\zeta_0}}{(7200)^2}.$$

The solution of the problem in hand is therefore contained in the following equations:

$$\left.\begin{aligned}
\frac{dr_0}{d\zeta_0} &= a' \sec^2\zeta_0 \sin 1'' + \tan\zeta_0 \frac{da'}{d\zeta_0} \\
n &= \frac{r - r' - (7200)\frac{dr_0}{d\zeta_0}}{(7200)^2} \\
k_1 &= \frac{a''}{1 + a''(1 + \sec^2\zeta_0)} \\
M &= k_1 \tan^2\zeta_0 \cos^2(\pi - q) \\
A &= +\tfrac{1}{2}M^2 \tan^2(\pi - q) \\
B &= +n[\cos^3(\pi - q) + \cos(\pi - q)\cot^2\zeta_0] \\
s - \sigma &= -\sigma(a + M) + A\sigma - B\sigma^2
\end{aligned}\right\} \quad (11)$$

The quantity ζ_0 may generally be arrived at by a single computation, based on the mean hour angle (t_0) and the mean declination (δ_0) of the two stars observed. But the value of ζ_0

* This method has been employed in preference to the computation of $\frac{d^2r_0}{d\zeta_0^2}$ by a formula derived by differentiation from (10) in consequence of the difficulty of obtaining an accurate value of $\frac{d^2a}{d\zeta_0^2}$ from the vertical refraction tables.

so obtained will not rigorously agree with the mean of the zenith distances of the two stars. A slight error may be introduced into the refraction at large zenith distances, unless ζ_0 be taken as the mean of the two zenith distances, separately computed. The following considerations show when it is sufficiently accurate to employ ζ_0 obtained from a single computation.

Since, in general,

$$\zeta = f(t, \delta)$$

$$\Delta\zeta = \left(\frac{d\zeta}{dt}\right)\Delta t + \tfrac{1}{2}\left(\frac{d^2\zeta}{dt^2}\right)\Delta t^2 + \ldots + \left(\frac{d\zeta}{d\delta}\right)\Delta\delta + \tfrac{1}{2}\left(\frac{d^2\zeta}{d\delta^2}\right)\Delta\delta^2 + \ldots \qquad (a)$$

But,

$$\left(\frac{d\zeta}{dt}\right) = \sin q \cos\delta \qquad \left(\frac{d^2\zeta}{dt^2}\right) = \frac{\cos\delta \cos\phi \cos A \cos q}{\sin\zeta}$$

$$\left(\frac{d\zeta}{d\delta}\right) = -\cos q \qquad \left(\frac{d^2\zeta}{d\delta^2}\right) = \sin^2 q \cot\zeta.$$

If we substitute these values in (a), and put

$$\Delta\zeta = \zeta - \zeta',$$

we easily get to terms of the second order inclusive:

$$\frac{\zeta+\zeta'}{2} = \zeta' + \tfrac{1}{2}\sin q_0 \cos\delta_0 (t-t') - \tfrac{1}{2}\cos q_0 (\delta - \delta').$$

Now, if ζ_0 be the zenith distance corresponding to

$$t_0 = \tfrac{1}{2}(t+t'),$$

we can obtain from (a), by putting

$$\Delta\zeta = \zeta_0 - \zeta'$$

to terms of the second order inclusive:

$$\zeta_0 = \zeta' + \tfrac{1}{2}\sin q_0 \cos\delta_0 (t-t') - \tfrac{1}{2}\cos q_0 (\delta - \delta')$$
$$- \tfrac{1}{8}\frac{\cos\delta_0 \cos q_0 \cos A_0 \cos\phi}{\sin\zeta_0}(t-t')^2$$
$$- \tfrac{1}{8}\sin^2 q_0 \cot\zeta_0 (\delta-\delta')^2.$$

So that we shall have the correction required by ζ_0:

$$d\zeta_0 = \frac{\zeta+\zeta'}{2} - \zeta_0 = \tfrac{1}{8}\frac{\cos\delta_0 \cos q_0 \cos A_0 \cos\phi}{\sin\zeta_0}(t-t')^2 + \tfrac{1}{8}\cot\zeta_0 \sin^2 q_0 (\delta-\delta')^2,$$

and, therefore,

$$d(\tan^2\zeta_0) = \tfrac{1}{4}\sec^2\zeta_0 [\sec\zeta_0 \cos\delta_0 \cos q_0 \cos A_0 \cos\phi (t-t')^2 + \sin^2 q_0 (\delta-\delta')^2],$$

and finally,

$$d(\text{refraction}) = \tfrac{1}{4}\sigma k_1 \cos^2(\pi - q)\sec^2\zeta_0 [\sec\zeta_0 \cos\delta_0 \cos q_0 \cos A_0 \cos\phi (t-t')^2$$
$$+ \sin^2 q_0 (\delta-\delta')^2].$$

It would lead to complicated expressions if we were to determine the maximum of this expression. We shall, however, certainly exceed this maximum if we put

$$d\,(\text{refraction}) = \tfrac{1}{4}\,\sigma^3 k_1 \sec^2\zeta_0\,(1+\sec\zeta_0)\sin^2 1'',$$

which amounts to $0''{\cdot}01$

for $\sigma = 4000''$ when $\zeta_0 = 76°$
,, $\sigma = 6000''$,, $\zeta_0 = 70°$
,, $\sigma = 8000''$,, $\zeta_0 = 60°$.

If, therefore, we observe very large distances far from the zenith, it will be advisable to compute ζ and ζ' separately, and employ

$$\zeta_0 = \tfrac{1}{2}(\zeta+\zeta').$$

The value of $s-\sigma$ given by equation (11) cannot theoretically be in error by $0''{\cdot}01$, even when the zenith distance is 80° and σ as large as $8000''$. But equation (11) gives the refraction correction corresponding to a mean state of the barometer and thermometer. In order to get its value for the actual state of the atmosphere it is necessary to employ in place of α'', in equation (11), the quantity κ, whose value is taken from the vertical refraction tables by means of the usual formula:

$$\kappa = \alpha'' \beta^{A''} \gamma^{\lambda''},$$

and instead of α, we must use

$$k = \alpha\beta^{A}\gamma^{\lambda}.$$

But for all practical purposes I think it will be found quite sufficient to employ the tables which I have prepared. These tables are founded on the constants of the Pulkowa refractions, and the example afterwards given for comparison has been computed with the Pulkowa tables.* I shall now explain the method used in the construction of the tables.

In equation (11) it is clear that the small term $B\sigma^2$ will not require any meteorological correction. I shall neglect the meteorological correction of the term $A\sigma$, since from the last term of equation (5*a*) we see that the meteorological correction of $A\sigma$ can never exceed

$$0''{\cdot}04\,\sigma\alpha''^2\tan^4\zeta_0,$$

or $0''{\cdot}006$ at 75° zenith distance, and $0''{\cdot}02$ at 80°.

There remains only the meteorological correction of the quantity

$$-\sigma\,(\alpha+\mathrm{M}),$$

which quantity may be called the "mean refraction."

* O. Struve, *Tabulæ Refractionum in usum Speculæ Pulcovensis congestæ*, Petropoli, 1870.

It will be convenient to so arrange matters that the meteorological correction may be found by simply multiplying the quantity

$$-\sigma(\alpha+\mathrm{M})$$

by a *meteorological factor*, to be taken from the tables, and adding the product to

$$-\sigma(\alpha+\mathrm{M}).$$

For this purpose let

f = the proper meteorological factor for k,

$f+x$ = the proper meteorological factor for α.

Then we shall have :

$$\text{Meteorological correction} = -\sigma(\alpha+\mathrm{M})f-\sigma x\alpha. \tag{12}$$

Now suppose that, in consequence of meteorological changes, k_1 becomes k_1+dk_1. It also becomes $k_1(1+f)$. Therefore

$$k_1+dk_1 = k_1(1+f),$$

or

$$f=\frac{dk_1}{k_1}. \tag{13}$$

Differentiating the third of equations (11) we get

$$\frac{dk_1}{d\alpha''}=\left(\frac{k_1}{\alpha''}\right)^2,$$

which substituted in (13) gives :

$$f=\frac{k_1}{\alpha''^2}d\alpha''.$$

But with the usual notation of the vertical refraction tables,

$$d\alpha''=\alpha''(\beta^{A''}\gamma^{\lambda''}-1).$$

So that, finally,

$$f=\frac{k_1}{\alpha''}(\beta^{A''}\gamma^{\lambda''}-1).$$

Similarly,

$$f+x=\beta^{A}\gamma^{\lambda}-1.$$

In both of these equations we shall produce no appreciable error if we write unity for A and A''. So we have

$$f=\frac{k_1}{\alpha''}(\beta\gamma^{\lambda''}-1) \tag{14}$$

$$f+x=\beta\gamma^{\lambda}-1, \tag{15}$$

from which we easily obtain, by subtraction,

$$x=\frac{k_1}{\alpha''}-1-\beta\gamma^{\lambda}\left(\frac{k_1}{\alpha''}\gamma^{\lambda''-\lambda}-1\right)$$

I have neglected the last term of this equation. Its maximum effect on $s-\sigma$ cannot exceed $0''{\cdot}004$ at zenith distance 70°, and $0''{\cdot}015$ at 80°. We may therefore write

$$x=\frac{k_1}{\alpha''}-1$$

and

$$-\sigma x\alpha=-\sigma\alpha\left(\frac{k_1}{\alpha''}-1\right).$$

This equation gives us the value of the last term of equation (12), and it may be incorporated with the last of equations (11) without producing any appreciable error. To do this we introduce K, defined by

$$\mathrm{K}=\alpha+\alpha\left(\frac{k_1}{\alpha''}-1\right)=k_1\frac{\alpha}{\alpha''},$$

and the last of equations (11) becomes

$$s-\sigma=-\sigma(\mathrm{K}+\mathrm{M})+\mathrm{A}\sigma-\mathrm{B}\sigma^2-\sigma(\mathrm{K}+\mathrm{M}) \qquad (16)$$

after we have added the meteorological correction.

In order to tabulate f I proceed as follows:—
Since λ'' can never differ much from unity, and since

$$f=f_{\lambda''=1}+\left(\frac{df}{d\lambda''}\right)_{\lambda''=1}(\lambda''-1),$$

we have from equation (14)

$$f=\frac{k_1}{\alpha''}(\beta\gamma-1)+\frac{k_1}{\mu\alpha''}\beta\gamma\log\gamma(\lambda''-1),$$

where μ is the modulus of the common system of logarithms; and if we put

$$f_0=\beta\gamma-1$$

$$\mathrm{Q}_0=\frac{k_1}{\mu\alpha''}\log\gamma(\lambda''-1),$$

this equation becomes

$$f=\frac{k_1}{\alpha''}f_0+\mathrm{Q}_0+f_0\mathrm{Q}_0. \qquad (17)$$

If we give to $\frac{k_1}{\alpha''}$ the constant value 0·995, we shall produce the following maximum errors in the refraction:—

At	45°	zenith distance	0″·002
„	60	„	0·004
„	70	„	0·005
„	80	„	0·025

We might tabulate f on the supposition that the indications of the attached and external thermometers are the same. But, if such is not the case, put

$t-\mathrm{T}$ = excess of attached over external thermometer in Fahr. degrees.

y = the increase in log β per degree Fahr. $= -0{\cdot}0000386$,

then f will require the correction

$$df = 0{\cdot}995\,\frac{y}{\mu}\,(f_0+1)\,(t-\mathrm{T}),$$

or with sufficient accuracy,

$$df = -0{\cdot}000089\,(t-\mathrm{T}),$$

so that (17) becomes

$$f = 0{\cdot}995\,f_0 + \mathrm{Q}_0 + f_0\,\mathrm{Q}_0 - 0{\cdot}000089\,(t-\mathrm{T}). \qquad (18)$$

It will be found convenient for computers to have f always positive. If R be the mean refraction, and f the meteorological factor, and if we put

v = an auxiliary number,

we have

$$v\mathrm{R} + v\mathrm{R}\left(\frac{1}{v}f + \frac{1-v}{v}\right) = \mathrm{R}\,(1+f) = \text{actual refraction}; \qquad (19)$$

which shows that we may substitute for f the quantity

$$\frac{1}{v}f + \frac{1-v}{v},$$

provided we substitute the quantity $v\,\mathrm{R}$ for R. Now, if we select v such that

$$v < f + 1,$$

then

$$\frac{1}{v}f + \frac{1-v}{v} > 1.$$

For convenience in the computation of the tables I put

$$v = 0{\cdot}82916 \text{ and } \frac{1-v}{v} = 0{\cdot}20604.$$

Equation (18) then becomes

$$\text{Met. factor} = 1{\cdot}2\,(\beta\gamma - 1) + 0{\cdot}20604 + [0{\cdot}0814]\,\mathrm{Q}_0 + [0{\cdot}0814]\,\mathrm{Q}_0\,(\beta\gamma - 1) - 0{\cdot}0001\,(t-\mathrm{T}), \qquad (20)$$

and the refraction, uncorrected for meteorological changes, will become, by (11) and (16),

$$\left.\begin{aligned} v\mathrm{M} &= vk_1 \tan^2\zeta_0 \cos^2(\pi - q) \\ s - \sigma &= -\sigma\,(v\mathrm{K} + v\mathrm{M}) + \mathrm{A}\sigma - \mathrm{B}\sigma^2 \end{aligned}\right\} \qquad (21)$$

$$\text{Log } \nu K \text{ and log } \nu k_1$$

may be taken from Table I. with the argument log $\tan^2 \zeta_0$.

$$1000\,A \text{ and } 1000000\,B$$

may be taken from Tables I. A. and I. B. They need only be multiplied by $\frac{\sigma}{1000}$ and $\left(\frac{\sigma}{1000}\right)^2$ to give the terms $A\sigma$ and $B\sigma^2$.

$$1{\cdot}2\,(\beta\gamma - 1) + 0{\cdot}20604$$

may be taken from Table II., with the arguments Barometer in English inches and Thermometer in Fahrenheit degrees.

$$[0{\cdot}0814]\,Q_0$$

may be taken from Table II. A. with the arguments log. $\tan^2 \zeta_0$ and "external thermometer." It is called the "first correction of the meteorological factor."

$$[0{\cdot}0814]\,Q_0\,(\beta\gamma - 1)$$

may be taken from Table II. B., using as arguments the quantities derived from Table II. and Table II. A.

$$-0{\cdot}0001\,(t - T)$$

may be taken, if desired, from Table II. C.

Although the tables are intended primarily for the computation of refractions when the *true* position angle and distance are known, they may also be used when we have only the observed distance and position angle. We can easily show that if we put

$$N = -\frac{s - \sigma}{\sigma},$$

then

$$\sigma - s = sN + \frac{s^2N^2}{s}$$

with an error that cannot exceed $0''{\cdot}002$ even at 80° zenith distance. But sN is the refraction computed by the tables, except that the apparent distance s has been used in place of the true distance σ. It is therefore evident that we can employ the apparent distance in computing the refraction, provided we apply to the result the correction

$$\frac{s^2N^2}{s}, \text{ or } \frac{[\text{Refraction}]^2}{\text{Distance}}.$$

This latter quantity has been tabulated, and may be taken from Table III. with the arguments apparent distance and refraction.

The position angle used should be the true position angle; so that if we wish to correct an observation of both position angle and distance, as in the case of a planet and comparison star, we would proceed as follows:—

I. Correct the observed position angle, which does not require a knowledge of the distance.
II. Compute the refraction in distance, using the true position angle from I. and the observed distance. Then add the correction from Table III.

If a series of stellar parallax observations are to be corrected for refraction in distance, it will be found convenient to arrange the work according to the following scheme. The same procedure is convenient whenever several measures of the same pair of stars are to be corrected.

I. Compute the true position angles and distances, using meridian observations of the principal star and the comparison stars. If meridian observations are not available, the true position angle and distance can be derived from one or two of the observations with sufficient accuracy.
II. Compute for every 10^m of hour angle, and for the mean declination of the two stars, the quantities q and $\tan^2 \zeta_0$, the computation being, of course, limited to hour angles corresponding to the observations. The quantities q and $\tan^2 \zeta$ can be tabulated once for all with the arguments declination and hour angle, and for the latitude of the observatory.
III. With the materials of I. and II. compute the mean refraction by equation (21) for every 10^m of hour angle, and tabulate it for every 10^m of sidereal time.*
IV. To obtain the proper refraction correction for any observation, enter the table computed under III. and take out the mean refraction. Take out the meteorological factor from the tables already explained (this factor will probably be constant for all the observations of a given night), and multiply the mean refraction by it. This gives the meteorological correction, which has only to be added to the mean refraction to get the actual refraction correction of the observation.

* If we are working at zenith distances large enough to make the terms $A\sigma$ and $B\sigma^2$ appreciable, this table should also give their sum. This sum should be placed in a separate column, as it does not need any meteorological correction. Log $\tan^2 \zeta_0$ should also be given, roughly, to serve as an argument for Table II. A.

I shall now take up the investigation of a formula for correcting position angles. If we substitute the values already given for a and b in equation (1) we have:

$$l_0 - \lambda_0 = \frac{\alpha'' \tan^2 \zeta_0 \sin 2\lambda_0}{2(1+\alpha)+\alpha'' \tan^2 \zeta_0} + \frac{1}{2}\frac{\alpha''^2 \tan^4 \zeta_0 \sin 4\lambda_0}{[2(1+\alpha)+\alpha'' \tan^2 \zeta_0]^2} + \ldots . \qquad (22)$$

If we substitute α'' for α in this equation the error cannot exceed $0''.2$ in $l_0 - \lambda_0$, and, if we omit the second term, we shall not introduce a greater error than $1''.6$ at 80°, or $0''.1$ at 70°. Making these changes, we find

$$l_0 - \lambda_0 = \frac{\alpha'' \tan^2 \zeta_0 \sin 2\lambda_0}{2(1+\alpha'')+\alpha'' \tan^2 \zeta_0} = \frac{\alpha'' \tan^2 \zeta_0 \sin 2\lambda_0}{2+\alpha''(1+\sec^2 \zeta_0)},$$

which, by means of equation (5), becomes

$$l_0 - \lambda_0 = \frac{\alpha''}{1+\frac{\alpha''}{k_1}} \tan^2 \zeta_0 \sin 2\lambda_0,$$

and if we give to $\frac{\alpha''}{k_1}$ the constant value 1·005, we have

$$l_0 - \lambda_0 = [9.69789]\, \alpha'' \tan^2 \zeta_0 \sin 2\lambda_0 \qquad (23)$$

with errors as follows:

At 80°	zenith distance	... ...	1″.6
„ 70	„	... ...	0·2
„ 60	„	... ...	0·6
„ 50	„	... ...	0·1

If we let F be the meteorological factor for equation (23), we shall have

$$F = \beta\gamma\lambda'' - 1,$$

or by (14)

$$F = \frac{\alpha''}{k_1} f;$$

and if we put

$$F = 1.005 f \qquad (24)$$

we shall only introduce the following insignificant errors in the refraction:

At 80°	zenith distance	... ...	1″.2
„ 70	„	... ...	0·2
„ 60	„	... ...	0·0
„ 50	„	... ...	0·1

It will be convenient to arrange matters so as to use the same meteorological factor for position angles and distances. We shall then be able to derive it from the tables already given.

Now, if R be the refraction in position angle, and V and c two constants, we can use in place of F the quantity

$$\frac{1}{V}F + \frac{1-V}{V} + c,$$

provided we substitute $(1-c)\,V\,R$ for R.
For

$$(1-c)VR + (1-c)VR\left[\frac{1}{V}F + \frac{1-V}{V} + c\right] = RF + R - cVR\left[\frac{1}{V}F + \frac{1-V}{V} + c\right]$$

$$= \text{actual refraction very nearly,}$$

since the quantity

$$-cVR\left[\frac{1}{V}F + \frac{1-V}{V} + c\right]$$

only has the following maximum values:

At 80° zenith distance	...	...	3″·6
„ 70 „	...	...	0·8
„ 60 „	...	...	0·3

Now, if t be the tabular meteorological factor for distances, we have by (19) and (24)

$$t = \frac{1}{\nu}f + \frac{1-\nu}{\nu} = [0{\cdot}07919]\,F + 0{\cdot}20604,$$

so that if t is to be also our meteorological factor for position angles, we must have

$$\text{Log}\,\frac{1}{V} = 0{\cdot}07919 \qquad \frac{1-V}{V} + c = 0{\cdot}20604,$$

and, therefore,

$$\text{Log}\,V(1-c) = 9{\cdot}91819.$$

So that equation (23) becomes, after multiplying by cosec 1″ and introducing the term depending on the declination,*

$$p - \pi = [5{\cdot}23154]\,\alpha''\tan^2\zeta_0 \sin(\pi - q)\cos(\pi - q) + [5{\cdot}23262]\,k'\tan\zeta \sin q \tan\delta_0. \qquad (25)$$

Equation (25) may now be used with the meteorological factor from Table II., and the quantities $[5{\cdot}23154]\alpha''$ and $[5{\cdot}23262]k'$ can be taken from Table I. with the argument $\log\tan^2\zeta_0$.

It is to be remarked that the meteorological factors for the two terms of equation (25) are not strictly equal. They differ by the quantity

$$\frac{1}{V}\frac{\beta\gamma\log\gamma}{\mu}(\lambda' - \lambda''),$$

which cannot, however, influence the refraction appreciably,† and is neglected. Furthermore, equation (25) requires the use

* See Chauvenet, *Spherical and Practical Astronomy*, vol. ii. p. 457.
† Unless, of course, the declination be very great.

of the true position angle, π, in computing the refraction. We can, however, employ the observed position angle, p, without producing an error greater than

$$[5{\cdot}23154]\,\alpha''\,(\pi-p)\tan^2\zeta_0\cos z\,(\pi-q)\sin 1'',$$

which will not in practice exceed $1''$ at 80° zenith distance.

I shall now give an example of the use of the tables. Suppose we have given

Barom. = 30·62 in. Therm. = 77° Fahrenheit

$\delta_0 = -20°\ 0'\ 0''$ $t_0 = 5^h\ 5^m$

$\pi = 164°\ 33'\ 36''$ $\sigma = 7200''$

$\phi = -33°\ 56'\ 3''{\cdot}2$

= latitude of Cape Observatory.

With these data we easily compute (or in the present case take from the manuscript tables of the Cape Observatory)

$\text{Log}\tan^2\zeta_0 = 0{\cdot}7828$ $q = 119°\ 33'\ 36''$.

So that

$$\pi - q = 45°\ 0'\ 0''.$$

The computation may now be arranged as follows:

For Distance.			For Met. Factor.		For Position Angle.	
Log σ	3·8573		Tab. II.	0·1770	Log [5·23] α'' (Tab. I.)	1·6659
Log υK (Tab. I.)	6·3606		II. A.	−0·0015	Log $\tan^2\zeta_0$	0·7828
Log $\sigma\upsilon$K	0·2179	No. $1''{\cdot}651$	II. B.	0·0000	Log $\sin(\pi-q)$	9·8495
			II. C.	0·0000	Log $\cos(\pi-q)$	9·8495
Log υk_1 (Tab. I.)	6·3521		f	0·1755	Log 1st term	2·1477
Log $\tan^2\zeta_0$	0·7828				1st term	$140''{\cdot}5$
Log $\cos^2(\pi-q)$	9·6990					
Log υM	6·8339				Log [5·23] k' (Tab. I.)	1·6747
Log σ	3·8573				Log $\tan\zeta_0$	0·3914
Log $\sigma\upsilon$M	0·6912	No. $4''{\cdot}912$			Log $\sin q$	9·9394
					Log $\tan\delta_0$	9·5611 n
Mean refraction		$+6''{\cdot}563$			Log 2nd term	1·5666 n
Aσ (Tab. I. A)		0·000			2nd term	$-36''{\cdot}8$
+ Bσ^2 (Tab. I. B)		+0·016			Refraction	$103''{\cdot}7$
Met. corr. (6·56 × 0·1755)		+1·152			Met. cor. (103·7 × 0·1755)	18 ·2
Refraction ($\sigma - s$)		$+7''{\cdot}731$			Refraction	$121''{\cdot}9$
						or $2'\ 1''{\cdot}9$

It will be interesting to compare the results of the preceding example with a rigorous trigonometrical solution. From the given values of σ and π we deduce

$$\tfrac{1}{2}(\alpha'-\alpha) = 1019''{\cdot}887 \qquad \tfrac{1}{2}(\delta'-\delta) = -3470''{\cdot}106,$$

which, being applied to t_0 and δ_0, give for the two stars

$$t = 76^\circ\ 31'\ 59''{\cdot}887 \qquad \delta = -19^\circ\ 2'\ 9''{\cdot}894$$
$$t' = 75^\circ\ 58'\ 0''\ 113 \qquad \delta' = -20^\circ\ 57'\ 50''{\cdot}106,$$

from which we get azimuths and zenith distances as follows:

$$A = 80^\circ\ 52'\ 18''{\cdot}13 \qquad \zeta = 68^\circ\ 36'\ 33''{\cdot}12$$
$$A' = 79^\circ\ 20'\ 42''{\cdot}94 \qquad \zeta' = 67^\circ\ 11'\ 42''{\cdot}13.$$

From the Pulkowa refraction tables we get by successive approximation, taking into account the barometric and thermometric conditions, the following refractions corresponding to ζ and ζ':

$$r = 142''{\cdot}050 \qquad r' = 132''{\cdot}500;$$

and, therefore, the following apparent zenith distances:

$$z = 68^\circ\ 34'\ 11''{\cdot}07 \qquad z' = 67^\circ\ 9'\ 29''{\cdot}63.$$

We can now solve the triangle formed by the zenith and the apparent places of the stars, and we obtain

$$s = 7192''{\cdot}252 \qquad p-q = 45^\circ\ 20'\ 20''{\cdot}79 \qquad p'-q' = 44^\circ\ 45'\ 49''{\cdot}91.$$

We also obtain, from the apparent zenith distances and azimuths, the apparent parallactic angles as follows:

$$q = 119^\circ\ 55'\ 36''{\cdot}35 \qquad q' = 119^\circ\ 9'\ 29''{\cdot}45,$$

which, being applied to the values of $p-q$ and $p'-q'$ immediately preceding, give the apparent position angles

$$p = 165^\circ\ 15'\ 57''{\cdot}14 \qquad p = 163^\circ\ 55'\ 19''{\cdot}36,$$

or the mean position angle

$$p = 164^\circ\ 35'\ 38''{\cdot}25.$$

Comparing these values of s and p with the values of σ and π originally assumed, we have

$$s-\sigma = -7''{\cdot}748 \qquad p-\pi = 2'\ 2'{\cdot}25.$$

Tables computed from the formulæ given in this paper have been separately printed. Copies may be obtained from the Royal Observatory, Cape of Good Hope.

Royal Observatory, Cape Town:
1890, *July.*

Note on Hind's Variable Nebula in Taurus. By S. W. Burnham.

Not long since I found a new double star in the vicinity of Hind's supposed variable nebula, and took occasion when the measures of the new pair were finished to examine the place of the nebula with the 36-inch refractor. The atmospheric conditions were always favourable. The distance of the components of the double star referred to is only $0''.16$, and, therefore, it would have been useless to turn the telescope in that direction when the definition was inferior.

The nebula is easily found from the 8·7 mag. star, D.M. $+19°$, 704, which is 15^s *p* and $4'$ *s*. The place of the nebula, as given by Dreyer, on the authority of D'Arrest, is identical with that of D.M. $+19°$, 706, the magnitude of which was estimated by Argelander as 9·4; and this is T *Tauri* of the variable star catalogues. On October 11 this star was estimated to be about 11·5 mag., but subsequent examinations indicated that it should be placed lower in the scale. I measured the difference in declination between this and the brighter star 15^s preceding; but finding the two stars could be connected directly by using the lowest micrometer eye-piece, the angle and distance were also measured on two nights. The measures in declination are as follows:—

1890·775	Diff. decl.	$+243''.65$
·777		243·40

The direct measures are:—

			Mag.	
1890·775	43°·5	$336''.31$	8·3	11·5
·777	43·8	336·07	8·4	12·5

This small star, if it is a star, is placed within a very small condensed nebula. It is somewhat elongated in the direction of 151°·7. A rough reading of the wires gave $4''.4$ for the length of the nebula in this direction. It is less extended on the opposite side of the star or nucleus, with a shorter diameter of perhaps half that measured. It was examined with various powers, but it was impossible to say whether or not it had a disc like that of an ordinary star. If it were the bright nucleus of the small nebula, it would probably present the same stellar appearance. It will be noticed that this description of the nebula does not correspond with that in the early observations, where it was noted, when it was seen at all, as about $1'$ in diameter.

On October 15 I asked Mr. Barnard to examine this region with the large telescope. His great experience in work of this kind, and remarkable acuteness of vision in detecting extremely faint, diffused objects, which would escape the ordinary observer, are well known. After careful attention he was able to see an excessively faint, round nebula, about $\frac{3}{4}'$ from the one previously described, in the estimated direction of 185°. This faint nebulosity was about 40″ or 50″ in diameter, and apparently not connected with the variable, and was of nearly the last degree of faintness for the light-power of the large instrument. It is, perhaps, too faint for any other telescope. I should not have seen it independently. Neither Mr. Barnard nor myself on any occasion could see the slightest trace of the OΣ nebula (seen also at times by D'Arrest, Tempel, and others, but invisible in the Rosse reflector), which should be 15^s preceding Hind's, nor any nebula in the immediate vicinity. Subsequently we looked at the small nebula with the 12-inch telescope, by way of getting a better estimate of its magnitude, and found that it was a very faint object, and in appearance precisely like any ordinary star of that magnitude. The nebulous surrounding was completely lost with the smaller instrument. I do not think it can be any brighter now than 12·5 mag. of the scale used in my double-star observations.

On the night of November 1 Mr. Keeler examined the nebula with the small spectroscope attached to the 36-inch telescope, and found that it was probably of the usual gaseous type, although on account of the extreme faintness of the nebula only the principal line at λ5005 was visible. The spectrum of the nucleus could not be seen.

Nothing seems to be definitely known of the period of this variable, T *Tauri*, and it seems to have been as much neglected as the nebula. It should be easy to determine, at least approximately, the extent and time of its change, and it is to be hoped that variable-star observers will give this object systematic attention. An instrument of moderate size will answer this purpose. It is probably now not far from its minimum brightness, in which case the variation must have a range of at least three magnitudes. A large aperture will be necessary to ascertain whether the small nebula, which is now visible immediately about the nucleus or star, is also subject to any change. I hope to be able to settle this question hereafter by making occasional examinations with the 36-inch refractor.

Lick Observatory:
1890, *November* 8.

New Nebulæ. By W. F. Denning.

The following nebulæ were discovered in *Camelopardus* while comet-seeking with a reflector of 10 inches aperture. With the exception of Nos. 1 and 10, they were first seen with a power of 60, field 50′. The positions may be regarded as correct within about 2′ or 3′, and they have been determined by comparison with small stars, which were kindly identified for me by Dr. Copeland, the Astronomer Royal for Scotland:—

No.	Date of Discovery.	Position for 1890. R.A. h m s	N.P.D. ° ′	Description.
1	1889, Aug. 26	4 29 59	14 34·8	F.S, bM, ✶12 Np.
2	1890, Nov. 7	4 40 19	11 52·1	F.S.R.
3	1890, Oct. 19	4 46 38	21 50·2	F.S.R., bMN, F dou. ✶Sf.
4	1890, Nov. 16	5 50 7	9 29·0	vF, S.
5	1890, Nov. 9	6 11 45	6 58·1	F.S.R, mbM.
6	1890, Oct. 17	6 59 26	4 15·0	vF, vvS, 12′ SSf, NGC 2300.
7	1890, Nov. 7	7 8 52	9 52·6	vF, pS, 22′ SSf, NGC 2336.
8	1890, Sept. 14	7 23 24	4 30·0	F.S.E., 46′ Sf, NGC 2300.
9	1890, Sept. 8	8 21 37	3 52 6	pF, S, mbM, ✶Nf.
10	1890, Aug. 23	8 34 30	4 5·6	F.S.R., gbM. In same field as preceding and Sff.

The place of No. 1 is probably very accurate, as it was given me by M. Charlois, of the Nice Observatory, who observed the nebula at my request.

Bristol: 1890, *November* 25.

Notes on Celestial Photographs taken at the Sydney Observatory. By H. C. Russell, B.A., F.R.S.

Since I sent you copies of some of my photographs of the Milky Way, &c., it has occurred to me that it was desirable that I should, with the same lens, &c., plate-development, and exposure, take a picture of some object well known to the members of the Royal Astronomical Society, so that there should be some measure of the work I have been doing upon southern objects. I therefore selected *Orion*, and upon an evening of average clearness I exposed a plate for four hours, and developed it as I did my other photographs. The glass positive and silver print from this which I have sent herewith will show better than any words what the photograph is like. Suffice it here to say that the

negative was much over-exposed, and shows folds of the nebula extending to the bright star ι. When developed, and before the intensifying solutions were applied, the negative was full of most beautiful detail of the central part of the nebula, but this, as will be seen, was all lost by intensifying the negative—in other words, treating it as my other negatives have been treated. The outer folds of the nebula shown here extend beyond those shown in any other photograph I have seen, but I have only seen some of the best. My photograph also shows some structure in the nebula about the third star in the Sword-handle, and if the effects here recorded are compared with those obtained with three hours' exposure on a good night upon η *Argûs*, it will afford a measure of the extreme faintness, photographically, of η *Argûs*.

I am also sending glass positive and silver-print reproductions from another negative of the *Nubecula Major*, obtained on October 17 with an exposure of 7 hours 3 minutes. This plate was taken upon the only available night since my previous letter, and brings out in a still more remarkable manner the spiral structure of this wonderful object, as well as two well-defined oval and spiral star clusters, the one about 3° north of the great one, the other south preceding it about the same distance. It is much to be regretted that the plate on which this negative was taken had several slight stains when developed; one occupies the whole of the preceding side of the plate, and can be traced past the preceding end of the great spiral; the others are small, faint, dark marks, extending across the plate and some distance from the centre.

Each increase in the time of exposure brings out new details of this striking object, all helping to indicate more and more clearly its spiral character. As seven hours is nearly the limit of darkness at this time of the year, I have placed the camera 18 miles out of the city, on a hill 615 feet high, and hope under a clearer atmosphere to secure a more perfect picture. I also send a print from a negative of *Nubecula Minor*, taken October 14 to 15, with eight hours' exposure, which brings out clearly the spiral character of this object, and its general similarity in form to the *Nubecula Major*.

Sydney Observatory:
1890, *November* 3.

Elements and Ephemeris of Zona's Comet.

By J. R. Hind, LL.D., F.R.S.

The following orbit depends upon an observation at Rome on November 16, one by Baron von Engelhardt on the 18th, and the Paris observations on the 21st :—

Perihelion passage, 1890, August 8·43592 G.M.T.

Longitude of perihelion	113° 16′ 52″·1	Appt. Eq. Nov. 20.
„ Ascending node	85 25 2·7	
Inclination	25 38 57·4	
Log perihelion distance	0·3138092	

Motion—retrograde.

The middle observation is thus represented $\Delta\lambda \cos\beta\,(c-o) = +0''\cdot3$, $\Delta\beta = +2''\cdot0$.

Ephemeris for Greenwich Midnight.

1890.	R.A. h m s	Decl. ° ′	Log Δ
Dec. 10	3 15 4	+33 57·7	0·2072
12	3 5 58	33 35·2	
14	2 57 26	33 11·1	0·2252
16	2 49 27	32 45·9	
18	2 42 0	32 19·9	0·2449
20	2 35 4	31 53·6	
22	2 28 38	31 27·4	0·2658
24	2 22 42	31 1·5	
26	2 17 14	+30 36·0	0·2872

The intensity of light on December 10 = 0·74, and on December 26 = 0·47, that on November 16 being taken as unity.

Ephemerides of the Satellites of Saturn, 1890–91.

By A. Marth.

(*Concluded.*)

Approximate Greenwich times of conjunctions of the satellites with the ends of the ring. In order to get the question settled whether the shadows of Tethys, Dione, and Rhea are discernible on the disc of *Saturn*, some of the times are added when the axes of the shadow-cones cross the central meridian.

1891.	h				h			h
May 16	8·8	Te. pn	May 22	8·6	Te. w	May 28	8·4	Te. ps
	10·6	Mi. pn		10·5	En. fs		10·8	Mi. fn
	11·5	Di. ps		12·2	Rh. w		14·5	En. pn
	15·6	En. ps		13·6	Mi. fs		15·4	Te. fs
	16·0	Mi. ps		16·4	Te. ps		16·7	Mi. pn
	16·1	Rh. fn	23	7·3	Te. e		19·1	Di. fn
	16·7	Te. w		7·7	Di. fn		19·3	Tit. inf. ♁
17	7·5	Te. fs		10·8	Rh. ps			n. 17″
	7·8	Di. e		12·3	Mi. fs	29	1·0	Tit. Shadow
	9·0	En. fn		12·4	Di. Shadow			near north limb
	9·2	Mi. pn		13·0	En. ps		7·0	Te. fn
	14·6	Mi. ps		15·1	Te. fn		9·4	Mi. fn
	15·3	Te. e		15·5	Di. pn		11·1	Te. Shadow
	15·4	En. pn		17·7	Mi. fn		14·0	Te. pn
18	13·2	Mi. ps		19·1	Te. Shadow		15·3	Mi. pn
	14·0	Te. w	24	10·9	Mi. fs		15·4	Di. w
	16·6	Di. w		11·8	En. pn		17·0	En. fn
	17·9	En. fn		13·8	Te. ps	30	9·5	En. ps
19	7·3	Rh. fs		16·3	Mi. fn		11·5	Rh. Shadow
	10·3	En. ps		16·6	Di. ps		11·8	Di. fs
	11·9	Mi. ps		18·4	Rh. e		12·7	Te. fs
	12·6	Te. e	25	9·5	Mi. fs		13·9	Mi. pn
	13·0	Di. fs		12·4	Te. fn		14·5	Rh. pn
	16·7	En. fs		12·9	Di. e		15·8	En. fs
20	9·1	En. pn		14·3	En. fn	31	8·3	En. pn
	10·5	Mi. ps		14·9	Mi. fn		8·4	Te. Shadow
	11·3	Te. w		16·5	Te. Shadow		11·4	Te. pn
	14·0	Di. fn		17·0	Rh. fn		12·5	Mi. pn
	16·4	Mi. fs	26	6·1	Di. Shadow		12·8	Di. fn
	18·7	Di. Shadow		6·8	En. ps		13·1	Rh. w
21	2·4	Tit. sup. ♁		8·1	Mi. fs		17·6	Di. Shadow
		s. 18″		9·2	Di. pn	June 1	9·1	Di. w
	8·1	Tit. close		11·1	Te. ps		10·0	Te. fs
		approach to		13·2	En. fs		10·8	En. fn
		shadow-cone		13·5	Mi. fn		11·2	Mi. pn
		of Saturn	27	9·7	Te. fn		11·7	Rh. ps
	9·1	Mi. ps		10·3	Di. ps		16·6	Mi. ps
	10·0	Te. e		12·1	Mi. fn		17·2	En. pn
	10·3	Di. w		13·8	Te. Shadow		17·9	Te. e
	10·5	Rh. Shadow		15·7	En. ps	2	8·7	Te. pn
	11·7	En. fn		16·7	Te. pn		9·6	En. fs
	13·6	Rh. pn	28	6·6	Di. e		9·8	Mi. pn
	15·0	Mi. fs		8·1	En. fn		15·2	Mi. ps
	17·8	Te. fn		8·3	Rh. fs		16·5	Te. w

1891.	h		h		h
June 2	18·0 Di. e	June 8	8·5 Te. w	June 17	7·4 Rh. fn
	19·3 Rh. e		12·4 Rh. Shadow		11·3 Te. pn
3	11·3 Di. Shadow		15·4 Rh. pn		13·4 Rh. Shadow
	12·1 En. ps		16·3 Te. ps		16·4 Rh. pn
	13·8 Mi. ps		17·9 Di. fn	18	7·9 Di. ps
	14·3 Di. pn	9	7·1 Te. e		9·9 Te. fs
	15·2 Te. e		14·1 Rh. w		15·0 Rh. w
	18·0 Rh. fn		14·3 Di. w		15·7 Di. fs
4	10·9 En. pn		15·0 Te. fn	19	8·6 Te. pn
	12·4 Mi. ps	10	10·6 Di. fs		13·7 Rh. ps
	13·8 Te. w		12·7 Rh. ps		16·4 Te. w
	15·4 Di. ps		13·6 Te. ps		16·8 Di. fn
5	11·0 Mi. ps	11	11·7 Di. fn	20	7·2 Te. fs
	11·7 Di. e		12·3 Te. fn		13·1 Di. w
	12·5 Te. e	12	8·0 Di. w		15·1 Te. e
	13·5 En. fn		10·9 Te. ps	22	1·0 Tit. sup. ♂
6	1·5 Tit. sup. ♂	13	9·6 Te. fn		s. 15″
	s. 16″		16·6 Te. pn		6·6 Tit. Middle
	7·4 Tit. Middle		16·8 Di. e		of eclipse
	of eclipse of un-		18·6 Tit. inf ♂	29	18·3 Tit. inf. ♂
	certain duration		n. 15″		n. 13″
	8·0 Di. pn	14	0·3 Tit. Shadow		23·5 Tit. Shadow
	9·2 Rh. fs		8·3 Te. ps	July 8	0·9 Tit. sup. ♂
	9·7 Mi. ps		13·2 Di. pn		s. 13″
	11·2 Te. w		15·3 Te. fs		5·9 Tit. Middle
	12·3 En. fs	15	10·2 Rh. fs		of eclipse
7	7·8 Rh. e		13·9 Te pn	15	18·3 Tit. inf. ♂
	9·1 Di. ps		14·2 Di. ps		n. 11″
	9·8 Te. e	16	8·8 Rh. e		22·2 Tit. Shadow
	16·9 Di. fs		10·6 Di. e		
	17·6 Te fn		12·6 Te. fs		

Approximate differences of right ascension and declination between Hyperion and the centre of Saturn.

Greenwich Noon.	$\alpha_7 - A$	$\delta_7 - D$		$\alpha_7 - A$	$\delta_7 - D$		$\alpha_7 - A$	$\delta_7 - D$
1891.	s	″		s	″		s	″
Jan. 7	−15·11	−22·9	Jan. 22	− 8·52	− 0·3	Feb. 6	+14·18	+19·6
8	12·47	−21·7	23	12·15	− 6·3	7	12·69	+21·8
9	8·89	−19·0	24	14·99	−11·9	8	9·73	+21·7
10	− 4·58	−14·7	25	−16·89	−16·8	9	5·74	+19·3
11	+ 0·14	− 9·1	26	17·74	−20·7	10	+ 1·20	+15·1
12	4·86	− 2·6	27	17·47	−23·4	11	− 3·45	+ 9·6
13	9·05	+ 4·3	28	16·08	−24·6	12	7·86	+ 3·4
14	+12·16	+10·7	29	−13·58	−24·3	13	11·72	− 3·1
15	13·76	+15·8	30	10·09	−22·2	14	14·80	− 9·5
16	13·62	+19·0	31	5·79	−18·3	15	−16·95	−15·4
17	+11·84	+19·9	Feb. 1	− 0·97	−12·8	16	18·03	−20·4
18	8·71	+18·7	2	+ 3·94	− 6·1	17	17·98	−24·2
19	4·69	+15·6	3	8·43	+ 1·3	18	16·78	−26·5
20	+ 0·21	+11·1	4	11·92	+ 8·7	19	−14·44	−27·1
21	− 4·33	+ 5·6	5	13·93	+15·0	20	11·06	−25·8

Greenwich Noon. 1891.	$\alpha_7 - A$	$\delta_7 - D$		$\alpha_7 - A$	$\delta_7 - D$		$\alpha_7 - A$	$\delta_7 - D$
	s	″		s	″		s	″
Feb. 21	− 6·82	−22·5	Mar. 25	− 1·73	+18·4	Apr. 26	− 4·78	−27·4
22	1·98	−17·3	26	6·28	+11·9	27	0·04	−21·4
23	+ 3·04	−10·5	27	10·37	+ 4·5	28	+ 4·69	−13·3
24	7·73	− 2·4	28	13·75	− 3·3	29	8·88	− 3·8
25	11·51	+ 5·9	29	−16·23	−11·0	30	11·99	+ 6·2
26	13·87	+13·7	30	17·69	−17·9	May 1	13·60	+15·4
27	+14·46	+19·8	31	18·04	−23·8	2	+13·49	+22·6
28	13·27	+23·6	Apr. 1	17·25	−28·3	3	11·76	+27·1
Mar. 1	10·51	+24·7	2	−15·32	−30·9	4	8·70	+28·5
2	6·63	+23·2	3	12·34	−31·4	5	4·76	+27·1
3	+ 2·10	+19·5	4	8·43	−29·6	6	+ 0·39	+23·1
4	− 2·62	+14·2	5	− 3·84	−25·4	7	− 3·98	+17·2
5	7·14	+ 7·7	6	+ 1·08	−18·8	8	8·09	+10·1
6	11·15	+ 0·6	7	5·88	−10·4	9	11·60	+ 2·2
7	14·37	− 6·5	8	10·00	− 0·7	10	14·33	− 5·7
8	−16·75	−13·4	9	12·88	+ 9·0	11	−16·16	−13·3
9	18·03	−19·5	10	+14·15	+17·7	12	16·97	−20·0
10	18·18	−24·4	11	13·63	+24·1	13	16·73	−25·5
11	17·17	−27·9	12	11·48	+27·8	14	15·42	−29·4
12	−15·08	−29·7	13	8·06	+28·3	15	−13·07	−31·1
13	11·80	−29·2	14	+ 3·84	+26·1	16	9·80	−30·6
14	7·68	−26·6	15	− 0·73	+21·5	17	5·77	−27·8
15	− 2·92	−21·8	16	5·24	+15·2	18	− 1·26	−22·5
16	+ 2·10	−15·0	17	9·34	+ 7·7	19	+ 3·34	−15·1
17	6·89	− 6·5	18	12·80	− 0·3	20	7·57	− 6·1
18	10·87	+ 2·7	19	−15·41	− 8·3	21	10·90	+ 3·5
19	13·52	+11·6	20	17·05	−15·8	22	12·85	+12·7
20	+14·45	+19·2	21	17·62	−22·3	23	+13·20	+20·2
21	13·57	+24·4	22	17·08	−27·4	24	11·94	+25·1
22	11·09	+26·9	23	−15·44	−30·8	25	9·31	+27·2
23	7·39	+26·5	24	12·56	−32·1	26	5·72	+26·4
24	+ 2·96	+23·5	25	9·12	−31·0	27	+ 1·61	+23·2